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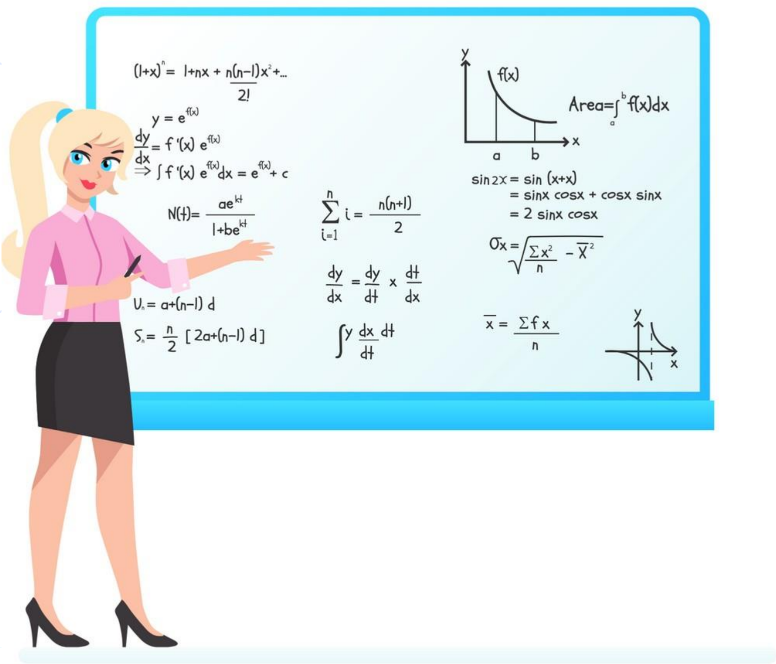
Abstract Algebra

Textbook and Course Advice



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Abstract Algebra:

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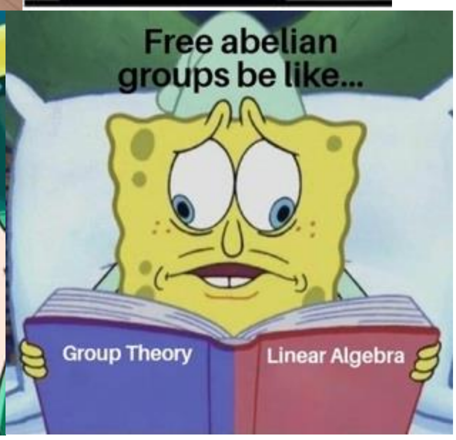

1 Pre-Requisites

Linear Algebra is the study of vector spaces and linear mappings between those spaces. Abstract Algebra deals with various abstract structures such as groups, rings and fields. A field is the "nicest" structure for the common people, since in a field all of the arithmetic and algebra that you have learned since elementary school "works." For example, in the field of real numbers you can solve the equation $2x = 3$ by multiplying each side of this equation by $1/2$. The same equation cannot be solved in the integers because the integers do not form a field. They do however form a ring. A ring has all the properties of a field except that nonzero elements do not need to have multiplicative inverses. This is precisely what prevents us from solving the previously mentioned equation.

Abstract algebra is on one hand a very self-contained subject. Everything can be defined abstractly, and you can prove interesting theorems without knowledge of anything else in mathematics. Knowing little bits of classical algebra, linear algebra, number theory and even calculus can help you to see some applications of what you learn, but studying the concepts by themselves does not require these things. However, something really nice about algebra is that you can study it in itself, but it also touches on nearly everything else in math in some way or another. Linear algebra is the prime example, but groups, rings and fields are also really common in other areas. When you eventually go on to study other more specific topics, you will find these things crop up again and again, so it will be good to have seen them at least once before.

I would say there aren't really any prerequisites at all, except the most frustrating and nebulous prerequisite of all: "mathematical maturity." Don't get me wrong, it helps to have seen some stuff: modular arithmetic helps, basic set theory helps, linear algebra helps, and even basic combinatorics helps (by "basic set theory," I mean stuff like equivalence relations, operations on sets like cross products, power sets, etc.). However, none of that stuff is strictly necessary. Most introductory abstract algebra books are self-contained from a logical point of view: they give you a few definitions, then push those around until you get a couple of lemmas, and eventually even a theorem or two. But at no point does a typical author invoke some fact from some other field (and if they do, it's typically in a very isolated example, and at most a handful of times in the book). Without mathematical maturity, the "hard" part isn't comprehending a particular definition or proof. Instead, the hard part is discerning any *feeling* for what's going on. Instead of a proper field of mathematics, the whole thing might look like a giant crossword puzzle with arbitrary rules.

The only way to develop skills with writing proofs is by experience. Abstract algebra is filled with enough interesting proofs to give you good examples for building intuition and plenty of challenging problems to keep you motivated. Some students first encounter proofs in linear algebra, and then in abstract algebra; it's a good domain of study for learning how to write proofs. It might be wise to get a hold of the book: "How to Prove It" by Daniel Velleman. In group theory, especially finite group theory, the basic theorems are all quite natural and accessible. As you delve into it, the material gets harder at a slow but steady rate, and the techniques and tricks used in the proofs also get more complicated. This is a good thing. It helps to broaden your mind and increase your sophistication.

2 Topic Intuition

2.1 Groups

Groups are abstract. They generalise a lot of different things from arithmetic, algebra, geometry and more. When studying arithmetic, you learn how to add, subtract, multiply and divide different kinds of numbers (Integers, fractions, real numbers, complex numbers). Then one day you suddenly realised that addition is subtraction in disguise. $7-4$ is in fact the same as $7+-4$ so instead of subtracting you're adding a negative number (subtraction is addition with inverses). Similarly, division is multiplication in disguise. $9 \div 5$ is the same as $9 \times \frac{1}{5}$. Division is multiplication with inverses (reciprocal). Instead of dividing you're really multiplying by a fraction. So, in arithmetic there are only really 2 operations, **addition** and **multiplication**. For addition opposites are negative numbers and for multiplication opposites are reciprocals. In abstract algebra we use the word "inverse" instead of "opposite". If you combine a number with its inverse, you get a special number called an identity element. For addition, the identity is 0. If you add 5 and its inverse -5 you get 0. What makes 0 unique is if you add it to any number, that number doesn't change; it retains its identity. For multiplication, the identity is 1. If you multiply a number by its inverse i.e. its reciprocal, you get 1. What makes 1 special? If you multiply it by any number, that number does not change. So, for both addition and multiplication we have numbers, we have inverses, we have identity elements, hence we have a group and are now ready to understand a group definition.

2.2 Fields

Ideally you should have knowledge of group theory and vector spaces before starting fields. The order normally taught is group theory, then fields and then vector spaces. Fields are taught before vector spaces, since in order to define a vector space we need to know what a field is

$\mathbb{F}$ is a set on which addition, subtraction, multiplication **and division** are defined and behave as the corresponding operations on rational and real numbers do i.e. set of numbers where you can add, subtract, multiply and divide.

A field is a mathematical object that on one hand is a relatively simple generalization of the ideas behind groups, but on the other will allow us to understand a variety of beautiful mathematical concepts and applications. We start here with the basics:
Common examples include
$\mathbb{R}$ Reals (decimals)
$\mathbb{C}$ (Complex numbers)
$\mathbb{Q}$ (Rationals)
$\mathbb{Z}$ is not a field since there is no multiplicative inverse for 2 or 3.
For example, we can't have $\frac{1}{2} \because \frac{1}{2} \notin \mathbb{Z}$

There are lots of other fields, even ones with only a finite number of elements called finite fields. Subtraction is the same as adding negatives and division is the same thing as multiplying by fractions, so we only need addition and multiplication. Subtraction and division are not separate composition laws.

2.3 Fields versus Groups

In a sense, a field is pretty much a set F that is a commutative group in two ways at the same time: that is, it is a group with respect to addition, and it is also a group with respect to multiplication if you ignore the additive identity 0! Every field is a group but not every group is a field. Fields require commutativity too and have 2 operations not just one!

A **group** has a SINGLE binary operation, usually called "multiplication" but sometimes called "addition", especially if it **is** commutative. A **field** has TWO binary operations, usually called "addition" and "multiplication". Both of them are always commutative. **Groups** model symmetries.

2.4 Rings

A ring is an additive commutative (aka abelian) group with an associative and distributive multiplication (aka abelian group under addition which also has multiplication). Essentially all the objects that students encounter in the first 12+ years of their maths education are elements of rings.
Fields have the operations of addition, subtraction, multiplication, and division that satisfy the expected properties, while rings only have three of those operations—**addition, subtraction, and multiplication**. There's nothing particularly difficult in understanding the general concept of ring, but you'll find that different kinds of rings (polynomial, quotient etc) have very different properties. When you're studying a particular kind of ring, keep in mind how that kind of ring is used. Know why people look at those rings in particular.

For a deeper more geometrical intuition into abstract algebra one should get an app called Brilliant (www.brilliant.org). There is a free section on group theory that explains them in a beautiful way!

Some examples to clarify difference between "rings", "groups" & "fields":

The set of integers $\mathbb{Z}$ under addition is a Group
The set of integers $\mathbb{Z}$ under multiplication is **not** a Group (no inverse for all members)

The set of integers $\mathbb{Z}$ under addition & multiplication is a Ring:
- It is an Abelian Group under addition
- Associative & distributive
- We do **NOT** need an inverse for multiplication

A Field is an Abelian Group under **BOTH** addition & multiplication
- The set of rational number $\mathbb{Q}$ is a field
- The set of integers $\mathbb{Z}$ is **NOT** a field (no inverse under multiplication)

3 Textbooks

3.1 Beginner

- **A Book of Abstract Algebra, Pinter**
  This is a wonderful introduction (a very accessible and gentle introduction)

- **Algebra In Action, Shahriari**

- **Schaum's Abstract Algebra**

- **A Transition To Abstract Mathematics, Maddox**

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- **A First Course In Abstract Algebra, Fraleigh**
  I highly recommend this book. It assumes very little and contains a lot of exercises. It is very intuitive, it has great exercises that help build competency with proofs, and the exercises are organized according to level of difficulty, beginning, e.g., with problems that are more computational, and building to more challenging proofs. In that sense, it is very developmental in its structure. The other thing worth noting is that it is "group" oriented.
  This also has a solutions manual which is great to help if you get stuck

- **Contemporary Abstract Algebra, Galian**
  This is a student friendly and concise textbook full of excellent examples, including only what you need to know for a first course - basically everything you could want in a first book. At the end it has a bunch of interesting topics that you probably won't see in a first-year course that are really cool to touch upon, since they are not any more advanced, just a little more niche.

- **Abstract Algebra, Hernstein**
  This is helpful and enjoyable to read. The proofs are elegant without being so terse as to make them difficult to parse; going through them helped me to develop a feel for proof-writing.The first chapter introduces fundamental concepts. Some of these will probably be new and others will probably be good to review from a more rigorous perspective: sets, functions, special types of functions, integers and their key properties, mathematical induction, and the basics of complex numbers. This is good material to be comfortable with, no matter where you head next in mathematics. The rest of the book focuses primarily on group theory, covering the fundamentals of the topic. After that, it includes a bit on theory of fields, abstract vector spaces, and polynomials, all important topics that you will see in greater depth later. The exercises are particularly good because (i) there are many of them; (ii) they are grouped by difficulty - make sure you understand all the easy ones, try at least a few of the more difficult ones; and (iii) they do a good job of introducing meaningful concepts, not simply providing busy work.

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  This contains lots of little gems in algebra and has lots of exercises and examples. While it does not assume the reader knows any algebra, it is a massive book, and can be a little intimidating. The style is sometimes a little more terse. All in all though, persistence can be really rewarding. It is used in many graduate schools for the preliminary/qualifying examination in graduate algebra, although many people would consider it an upper undergraduate book and is a like a bible to many students!

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  This is a mixture of all styles. It is the biggest, with around 1000 pages, it has an informal style in its explanations but a highly rigorous development, with insightful examples and filled also with lots of exercises (which I particularly find well-suited for self-study)

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